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FIXED POINT THEOREM FOR COMMUTING MAPPING

Piyush Kumar Tripathi[*1] & Alok Agrawal[2]
Amity University Uttar Pradesh
(Amity School Of Applied Sciences, Lucknow)

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ABSTRACT

It can be observed that completeness of a metric space is not enough to ensure the existence of fixed point for contractive mappings. So, fixed point theorems for such mappings require further restriction on the space or extra conditions have to be imposed on mappings or some restrictions imposed on its range. Edelstein had shown that compactness of the metric space (X,d) guarantees a unique fixed point for a contractive mapping on X . In this paper,the commutative maps are used as a tool for generalizing some of the results.

I. INTRODUCTION

A number of authors have defined various contractive type self mapping of metric spaces which are generalizations of well knownBanach contraction principle and have used the same technique. The contractive condition on maps produce suitable iterations, which give Cauchy sequence and a hypothesis of completeness in the range containing these sequences. These sequences produce a limit point, which becomes a fixed point of the mapping. The contractive condition on mapping has two roles; first they assure that certain iterations are Cauchy, and second, they assure the uniqueness of fixed point.

Some common fixed point theorems using sequence which are not necessarily obtained as a sequence of iterates of certain mappings are motivated by a result of Jungck [4]. He proved that a continuous self mapping f of a complete metric space (X,d) has a fixed point provided there exists $q \in (0,1)$ and a mapping $g: X \to X$ which commute with f and satisfies

(a) $g(X) \subseteq f(X)$

(b) $d(gx, gy) \leq qd(fx, fy)$, for all $x, y \in X$. Then g and f have unique common fixed point.

Some common fixed point theorems using sequence which are not necessarily obtained as a sequence of iterates of certain mappings are motivated by a result of Jungck [3]. He proved that a continuous self-mapping f of a complete metric space (X,d) has a fixed point provided there exists $q \in (0,1)$ and a mapping $g: X \to X$ which commute with f and satisfies

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(b) $d(gx, gy) \leq qd(fx, fy)$, for all $x, y \in X$. Then g and f have unique common fixed point.

The above theorem promoted the commutative maps as a tool for generalizing some of the results. Subsequently, using commuting- map concept, a variety of variations and generalizations of the above theorem were obtained by Yeh [12], Park [5], Singh and Pant [8]. In 1982, Sessa [6] introduce a generalization of the commuting map-concept by saying that maps $f, g: (X,d) \to (X,d)$ are weakly commuting if $d(fgx, gfx) \leq d(fx, gx)$ for $x \in X$.

This property is strictly weaker than the commutativity. In 1986, Jungck [4] introduced the concept of compatibility, which is weaker than weakly commutativity. The above concept of S. Sessa [6] was generalized by Singh and Pant [8] by introducing the definition of weakly commuting mappings in probabilistic metric space. In the sequel Dimri and Gairola [2] defined R-weakly commuting mappings in probabilistic metric space

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and proved some common fixed point theorems.

The above developments motivated Piyush Kumar Tripathi [11] to introduce the definition of generalized R-weakly commuting mappings in probabilistic metric space, generalizing the definition of R- weakly commuting mappings defined by Dimmri and Gairola [2] and Singh and Pant [8]. As a consequence of this definition, Piyush Kumar Tripathi [11] proved some common fixed point theorems using a lemma of Singh and Pant [8].

DEFINITION [8]: Two self mappings *f* and *g* on a probabilistic metric space *X* will be called weakly commuting if $F_{fgp,gfp}(x) \geq F_{fp,gp}(x)\ \forall\ p \in X$ and $x > 0$

In 1998 R.C. Dimiri and U.C. Gairola [2] revised the above definition of S.L. Singh and B.D. Pant and called R-weakly commuting mappings.

DEFINITION [2]: Two self mappings *f* and *g* on a probabilistic metric space *X* is said to R - weakly commuting if there exist a real number $R > 0$ such that $F_{fgp,gfp}(Rx) \geq F_{fp,gp}(x)\ \forall\ p \in X$ and $x > 0$.

Piyush Kumar Tripathi [11] defined generalized R-weakly commuting mappings as,

DEFINITION[11]: Two self mappings *f* and *g* on a probabilistic metric space *X* be called generalized R-weakly commuting if there exist a real number $R > 0$ such that

$$F_{fgp,gfq}(Rx) \geq F_{fp,gq}(x)\ \forall\ p,q \in X \text{ and } x > 0.$$

From the definition of generalized R- weakly commuting mappings we notice that generalized R- weakly commuting mappings implies R- weakly commuting mappings without holding converse.

Following is the useful lemma proved by S.L. Singh and B.D. Pant [8].

1. **LEMMA:** Suppose $\{p_n\}$ is a sequence in Menger probabilistic metric space (*X*,*F*,*t*), where *t* is continuous and $t(x,x) \geq x\ \forall\ x \in [0,1]$. If $\exists\ k \in (0,1)$ such that $\forall x > 0$ and positive integer *n*, $F_{p_n,p_{n+1}}(kx) \geq F_{p_{n-1},p_n}(x)$. Then $\{p_n\}$ is a Cauchy sequence.

II. MAIN RESULTS:

THEOREM: Suppose (*X*,*F*,*t*) be a complete Menger probabilistic metric space,

where $F_{p,q}$ is strictly increasing distribution function in (0,1), $\forall p \neq q$. Let *f* and *g* are two pair of self mappings on *X*, satisfying.

(a) f and g are contraction pair.

(b) $f(X) \subset g(X)$ and f is continuous.

(c) f and g are generalized R - weakly commuting mappings.

Then *f* and *g* have unique common fixed point.

PROOF: Let $p_0 \in X$, then as above theorem 3.2.1 we can construct a sequence $\{p_n\}$ such that $f(p_n) = g(p_{n+1})$. For $x > 0$,

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$$F_{fp_n, fp_{n+1}}(kx) \geq \min\{F_{fp_n, gp_{n+1}}(x), F_{fp_{n+1}, gp_{n+1}}(x), F_{gp_{n+1}, gp_n}(x), F_{fp_n, fp_{n+1}}(x)\}$$

$$F_{fp_n, fp_{n+1}}(kx) \geq \min\{F_{fp_n, fp_n}(x), F_{fp_{n+1}, fp_n}(x), F_{fp_n, fp_{n-1}}(x)\}$$

$$F_{fp_n, fp_{n+1}}(kx) \geq \min\{1, F_{fp_{n+1}, fp_n}(x), F_{fp_n, fp_{n-1}}(x)\}$$

$$F_{fp_n, fp_{n+1}}(kx) \geq \min\{F_{fp_n, fp_{n+1}}(x), F_{fp_{n-1}, fp_n}(x)\}$$

i.e. $F_{fp_n, fp_{n+1}}(kx) \geq F_{fp_{n-1}, fp_n}(x)$,

because $F_{p,q}$ is strictly increasing distribution function in (0,1), $\forall p \neq q$, so by lemma 2.1.1 $\{fp_n\}$ is a Cauchy sequence. Since (*X*,*F*,*t*) is complete, $fp_n \to z \in X$ i.e. $gp_n \to z$.

Since *f* is continuous so, $ffp_n \to fz \in X$ and $fgp_n \to fz$. Since *f* and *g* are generalized R – weakly commuting so $gfp_n \to fz$.

Now we try to provez is a common fixed point of *f* and *g*, first we prove that *z* = *fz*otherwise if $z \neq fz$. Then,

$$F_{fp_n, ffp_{n+1}}(kx) \geq \min\{F_{fp_n, gp_n}(x), F_{ffp_n, gfp_n}(x), F_{gp_n, gfp_n}(x), F_{fp_n, ffp_n}(x)\}, n \to \infty$$

$$F_{z, fz}(kx) \geq \min\{F_{z, fz}(x), F_{fz, fz}(x)\}$$

i.e. $F_{z, fz}(kx) \geq F_{z, fz}(x)$, which is possible because $F_{p,q}$ is strictly increasing distribution function in (0,1), $\forall p \neq q$, but in our case *kx*<*x*, so *z* = *fz*. Since $f(X) \subset g(X)$, so $\exists\, z_1 \in X$ such that $z = fz = gz_1$. Next we prove that $fz_1 = z$ otherwise if $fz_1 \neq z$ then,

Again,

$$F_{ffp_n, fz_1}(kx) \geq \min\{F_{ffp_n, gfp_n}(x), F_{fz_1, gz_1}(x), F_{gfp_n, gz_1}(x), F_{ffp_n, fz_1}(x)\}, n \to \infty$$

$$F_{fz, fz_1}(kx) \geq \min\{F_{fz, fz}(x), F_{fz_1, z}(x), F_{fz, gz_1}(x), F_{fz, fz_1}(x)\}$$

$$F_{z, fz_1}(kx) \geq \min\{1, F_{fz_1, z}(x), F_{fz, gz_1}(x), F_{fz, fz_1}(x)\}$$

$$F_{z, fz_1}(kx) \geq \min\{1, F_{z, fz_1}(x),\}$$

i.e. $F_{z, fz_1}(kx) \geq F_{z, fz_1}(x)$.

Which is not possible so $z = fz = fz_1 = gz_1$.

Again,

$F_{fz, gz}(Rx) = F_{fgz_1, gfz_1}(Rx) \geq F_{fz_1, gz_1}(x) = 1$, so $F_{fz, gz}(Rx) = 1 \Rightarrow fz = gz = z$.

Thereforez is common fixed point of *f* and *g*. For uniqueness suppose *z* and z' are two common fixed point of *f* and *g* i.e. $fz = z = gz$ and $fz' = z' = gz'$.

Now,

$$F_{z,z'}(kx) = F_{fz,fz'}(kx) \geq \min\{F_{fz,gz}(x), F_{fz',gz'}(x), F_{gz,gz'}(x), F_{fz,fz'}(x)\}$$

$$F_{z,z'}(kx) \geq \min\{1, F_{z,z'}(x)\}$$

i.e. $F_{z,z'}(kx) \geq F_{z,z'}(x)$.

Which is possible so $z = z'$.

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